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Gamma Process- Lifecycle analysis of the Neumarkt Bridge

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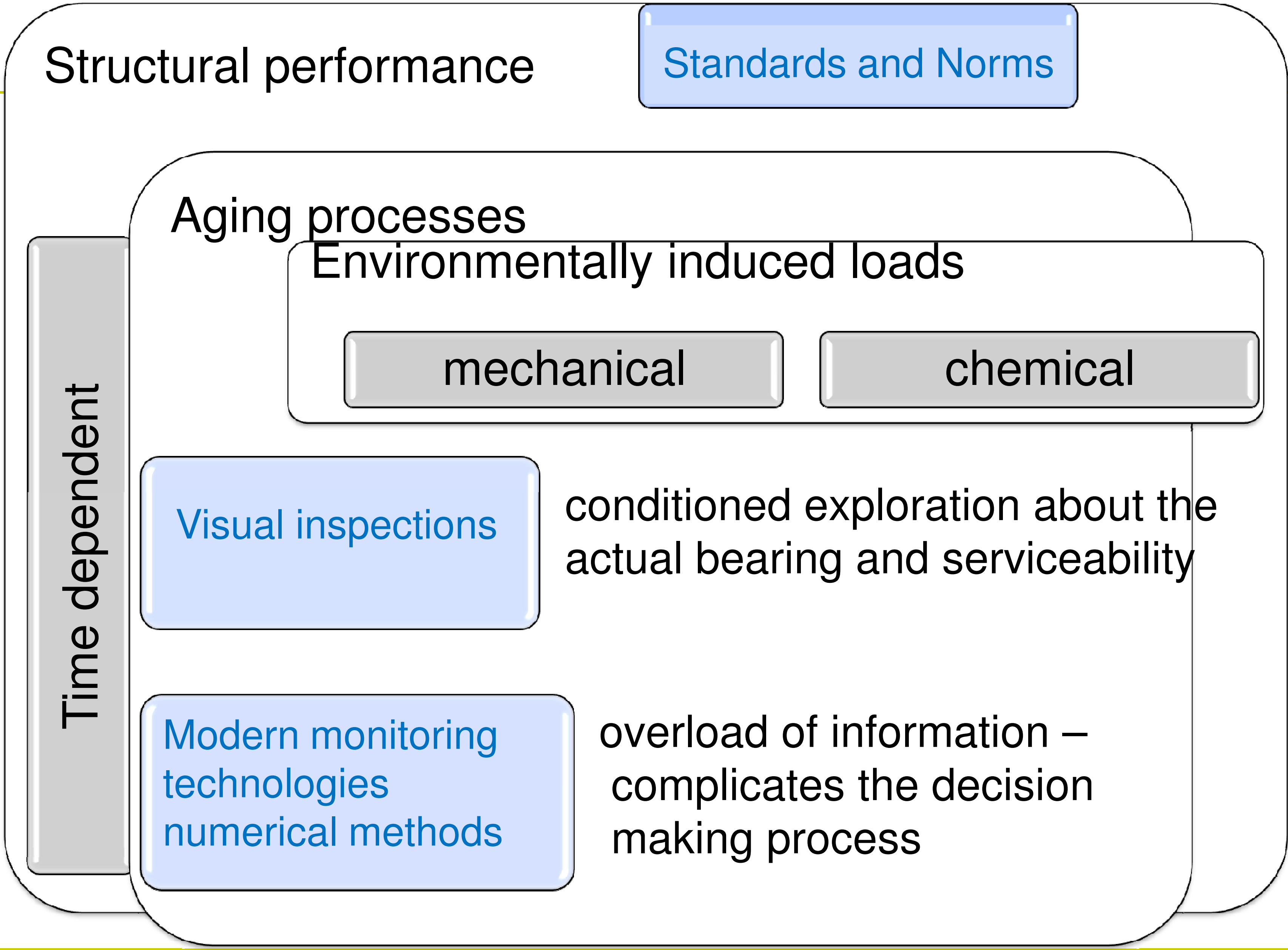
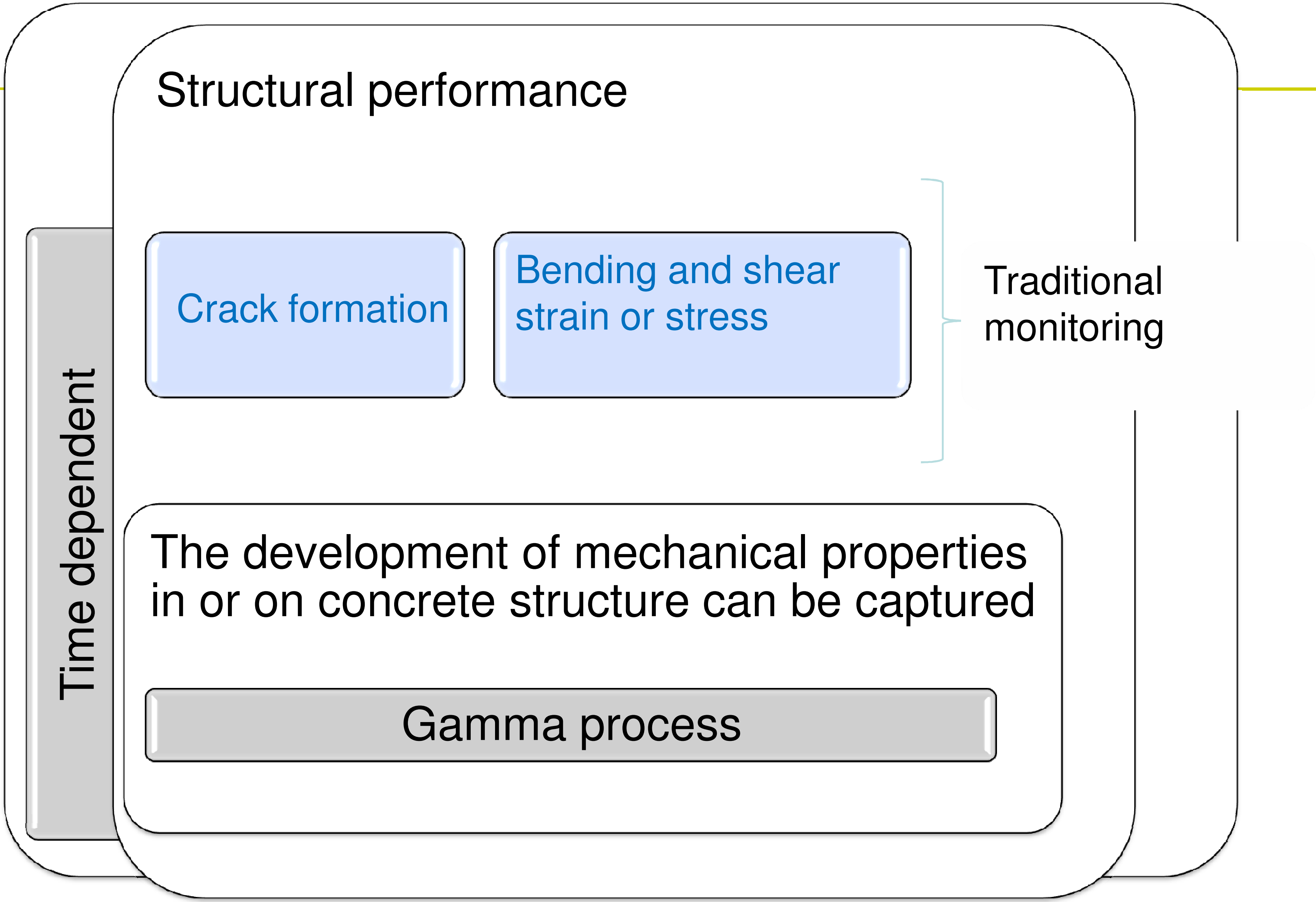

Structural performance
Standards and Norms
Time dependent
Aging processes
Environmentally induced loads
mechanical
chemical
Visual inspections
conditioned exploration about the actual bearing and serviceability
Modern monitoring technologies numerical methods
overload of information – complicates the decision making process


Structural performance
Crack formation
Bending and shear strain or stress
Traditional monitoring
The development of mechanical properties in or on concrete structure can be captured
Gamma process
Time dependent

Concrete lifetime degradation

Degradation model (Elligwood and Mori, 1997)

$$\alpha(t) = c\,(t - Ti)^{b}$$

t = time,

Ti = deterioration initiation time

c= rate parameter ,

b = time- order parameter

Model application

- for behavioural limit state of interest where the structural resistance can be related to $\alpha(t)$

Model parameters

•empirically determined from laboratory experiments

linear shape (b=1) for the corrosion of concrete reinforcement, parabolic (b = 2) for the sulphate attack, square root (b = 0.5) for the diffusion-controlled aging

Calibration of model parameters

structural inspection and monitoring techniques

Random variable approach

Uncertainties in structural systems

- load and material properties
- human actions
- model structures and parameters

- Several efforts are being made to include uncertainties in the life time assessment of concrete structures

  - Stochastic analysis ⟶ random variables approach

random variable degradation model

- The random path of deterioration remains fixed over the entire lifetime

- However the evolution of the process is uncertain over the lifetime of a structure

- The temporal uncertainty still remains pushed aside

Random process approach - recommended

Gamma process

- Uncertainties associated with the deterioration in lifetime are possibly expressed by gamma process

  - a continuous time stochastic process that begins with *0*

  - allows stationary independent jump of increment

- This continuous time stochastic process incorporates a number of ***gamma distributed increments***

Gamma Process

Characteristics

- the increments are independent
- gamma distributed
- non negative
- pure jump
- with a shape function > 0
- scale parameter > 0

- The probability density function:

$$Ga(x|\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \cdot x^{\alpha-1} \cdot \exp(-\beta \cdot x)$$

- where

$$\Gamma(\alpha) = \int_{z=0}^{\infty} z^{\alpha-1} \cdot e^{-z} \cdot dz$$

is the Gamma function for $\alpha > 0$

Modeling of Gamma process

- A continuous-time stochastic process $\{X(t),\ t \geq 0\}$ is characterized by independent increments

- Probability distribution function of $X(t)$, with the time variable t,

$$f_{X(t)}(x) = Ga\left(x, \alpha(t), \beta\right)$$

- The expected value

$$E\left(X(t)\right) = \frac{\alpha(t)}{\beta}$$

Modeling of Gamma process

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$$E(X(t)) = \frac{\alpha(t)}{\beta}$$

- Variance:

$$Var(X(t)) = \frac{\alpha(t)}{\beta^2}$$

$$COV(X(t)) = \frac{\sqrt{Var(X(t))}}{E(X(t))} = \frac{1}{\sqrt{\alpha(t)}}$$

Gamma process modeling of deterioration process

- Degradation model $$\alpha(t) = ct^{b}$$
- The deterioration rate X(t) at the time t, with t ≥ 0 can be described by:
  - the shape parameter $\alpha(t) = ct^{b}$
  - the scale parameter β

C = random rate of degradation (unknown)

β = *scale parameter (unknown)*

The unknown determined by using experts' judgment and statistic

Parameters estimation

Statistical methods

Maximum Likelihood Method	Method of Moments	Bayesian Statistics

Method of Moments

provides estimators by equating

- sample moments with the corresponding distribution moments (unobservable population moments)
- computing equations for the quantities to be estimated

Method of Moments

- Estimation of parameters $\hat{c}$ and $\hat{\beta}$:

$$\frac{\hat{c}}{\hat{\beta}} = \frac{\sum_{i=1}^{n} \gamma_i}{\sum_{i=1}^{n} w_i} = \frac{x_n}{t_n^b}$$

Method of Moments

- Estimation of parameters $\hat{c}$ and $\hat{\beta}$:

$$\frac{\hat{c}}{\hat{\beta}} = \frac{\sum_{i=1}^{n} \gamma_i}{\sum_{i=1}^{n} w_i} = \frac{x_n}{t_n^b}$$

$$\frac{\hat{c}}{\hat{\beta}} = \bar{\gamma} \cdot \frac{x_n}{\hat{\beta}} \cdot \left(1 - \frac{\sum_{i=1}^{n} w_i^{\,2}}{\left[\sum_{i=1}^{n} w_i\right]^2} \right) = \sum_{i=1}^{n} \left(\gamma_i - \bar{\gamma} \cdot w_i \right)^2$$

- where $w_i = t_i^b - t_{i-1}^b \quad , \quad \gamma_i = X_i - X_{i-1}$

Case Study (Neumarkt bridge)

- Crosses the A22, the Italian section of the Brenner highway,
  - between the provincial towns of Neumarkt and Auer

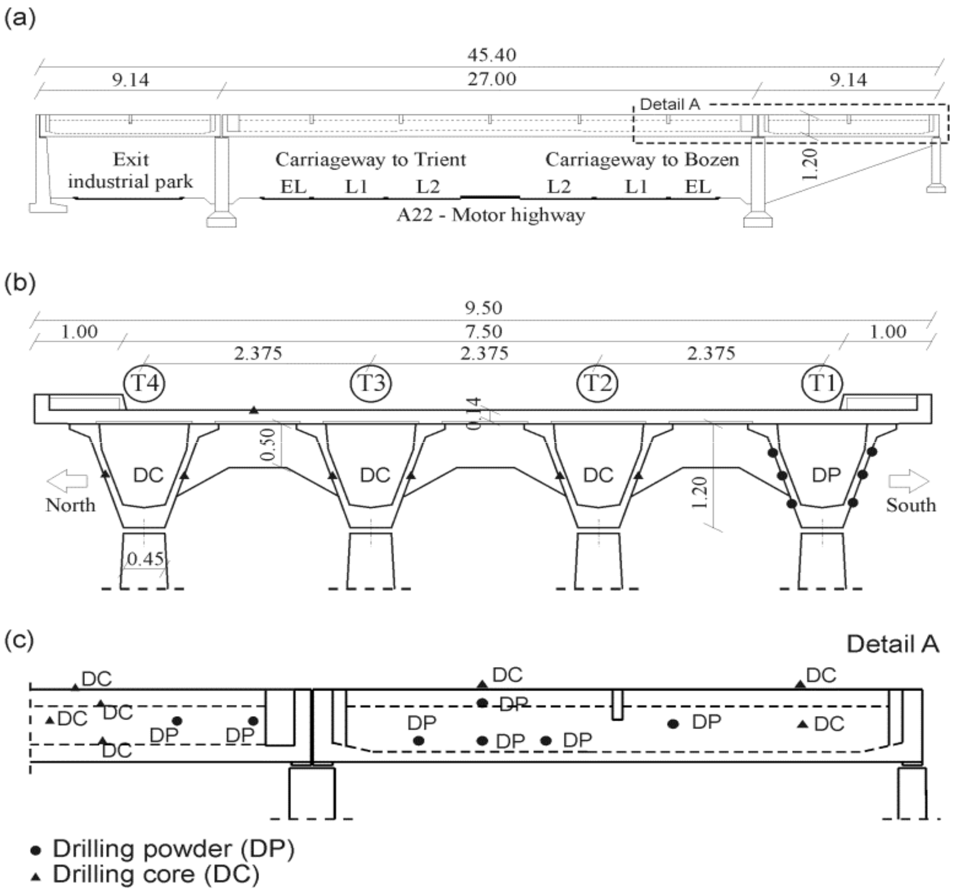
- A three-span bridge constructed from precast elements

- The four V-shaped precast elements mounted side by side carry a thin concrete slab

Statistical characteristics of the structural response

Characteristica	Time t_P	R		S(60% LM1)			S(83% LM1)		
	[Years]	Mean	COV	Mean	COV	β	Mean	COV	β
Deflection, u_z [m]	0 30 50	0.11 0.11 0.11		-0.07 -0.07 -0.08	0.04 0.04 0.04	+ + 8.7	-0.09 -0.10 -0.12	0.04 0.05 0.05	5.1 2.0 -
Crack width w [mm2]	0 30 50	0.20 0.20 0.20		0.07 0.05 0.01	0.17 0.20 0.11	+ + +	0.06 0.07 0.02	0.11 0.14 0.08	+ + +
Concrete stress σ [Mpa]	0 30 50	18.00 18.00 18.00		-14.12 -15.34 -17.10	0.01 0.01 0.01	+ + 7.5	-19.02 -20.25 -21.83	0.01 0.01 0.01	- - -
Bearing Capacity [load step]	0 30 50	48.30 45.10 39.00	0.08 0.01 0.06	20.0 20.0 20.0	7.3 5.6 8.1		30.0 30.0 30.0	4.7 3.3 3.8	

Geometry


(a)
45.40
9.14
27.00
9.14
Detail A
Exit
industrial park
Carriageway to Trient
Carriageway to Bozen
EL L1 L2
L2 L1 EL
A22 - Motor highway
1.20
(b)
9.50
1.00
7.50
1.00
2.375
2.375
2.375
T4
T3
T2
T1
0.14
0.50
1.20
DC
DC
DC
DP
North
South
0.45
(c)
Detail A
DC
DP
● Drilling powder (DP)
▲ Drilling core (DC)

Gamma process prediction of statistical characteristics of structural response; evaluated for 83% of the LM1 load model

Characteristica	Time tp	R		S(83% LM1)	
	[Years]	Mean	COV	β	C
Deflection, u_z [m]	0	0.09	0.04	-	-
	30*	0.10	0.04	**13.12**	0.044
	50	0.12	0.04	**34.22**	0.082
	35	0.08	0.08	1.764	0.006
	40	0.09	0.07	3.086	0.010
	50	0.12	0.06	5.967	0.020
	60	0.14	0.06	8.573	0.029
	70	0.16	0.05	10.87	0.036
	80	0.21	0.05	15.32	0.051
	100	0.23	0.05	17.49	0.058

Gamma process based condition assessment

- The evolution or progression of structural deterioration is modelled by gamma process
- Independent deterioration increments are characterized by with different shape and scale parameters
- Any estimation of parameters during the early years is best established by experts.

Thank you for your attention!